

**ASSIGNMENT-4**

**M.Sc. (Maths) 2<sup>nd</sup> Semester**

**Subject-Measure Theory and Integration**

If  $f \in L^2[0,1]$  show that  $|\int_0^1 f(x)dx| \leq (\int_0^1 |f(x)|^2 dx)^{\frac{1}{2}}$ .

1. Show that the space  $L^p$  is a normed linear space for  $1 \leq p < \infty$ .
2. State and prove Jensen's Inequality.
3. If  $f, g \in L^p$  ( $1 \leq p < \infty$ ) then  $f + g \in L^p$ .
4. Show that  $(L^p, d)$  is a metric space.
5. Prove that a sequence of function in  $L^p$  space has at most one limit.